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Application of quantitative metrics for assessing the investment risk

1. Introduction

The main objective of enterprises, financial institutions and commercial organizations is to generate profit. Entities shall make investments pending the return of a particular height. If these expectations shall have the real basis for implementation, they should use the risk analysis at the same time. This will allow to assess of whether this rate is possible.

This article will briefly described the theoretical aspect of the assessment of investment risk. Assessment investment risk can be based on quantitative and qualitative measures. **The purpose of this article** is to show the selected methods of quantitative risk assessment (variability, sensitivity, and downside risk measures), and to present a case study, together with the real possibilities of using these measures in practice of the organization. With the given assumptions the value for each measure will be calculated and its interpretation and meaning in practice will be given.

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2. Investment risk and its assessment

The assessment of the identified risk is an important task of the risk management.

The result of risk assessment provides information that is essential for the answers to two questions (see Zellmer, Wasilewski 2010, p. 398):

1. Is the implementation of actions that are related to risk acceptable from an economic and legal point of view?
2. To what extent must the measures to reduce the identified risk and , if necessary, to achieve the admissibility of actions be planned and implemented?

Risk assessment assumes that the notion of risk is defined in a precise and measurable way. The concept of risk is understood as 'the possibility of non-achievement of an explicitly formulated or implicitly arisen goal of an enterprise'. If the objective can be expressed quantitatively (e.g. amount of profit), then risk will be the likelihood that, that obtained value will be different than expected. (see Zellmer, Wasilewski 2009, p 15)

Investment risk is defined as the possibility that the realized rate of return may be different from the expected rate of return by an investor. This risk can be considered in two ways (Nahotko 1997, pp. 83-88):

- a single investment - the company assess the risk of the project without the relation with the total risk of the company,
- the company - in the case of new investments, manager consider the impact of this investment to the risk of companies.

3. Quantitative methods for risk assessment

3.1. Risk measures

Risk models use different metrics. All are widely used. They include (cf. Bessis 2010, p. 180):

- volatility measures,
- sensitivity measures,
- downside risk measures.

Volatility measures are measures of the magnitude of variations of an asset value or of its risk factors volatility is the second moment of probability distribution, or standard deviations . The absolute measure include: the variance, standard deviation, mean deviation, range. The relative measure of dispersion include: the variation coefficient, which specifies the size of the risk incurred in relation to profit.

Sensitivity measures are measure of the response of an asset value to a shock on the underlying market parameters, commonly called "risk factors". These measures include: beta coefficient, Greek coefficients, duration and many others.

Downside risk measures, more commonly, "value-at-risk" or "VaR": value-at-risk is the modeled value of potential losses in monetary value which synthesizes all risk metrics in a single potential loss figure. Often used measures are: Value at Risk, (semivariance of return, standard semideviation of return, mean semideviation, safety level, aspiration level.

Only selected measures will be discussed below, and then will be used in the case study (see chapter 4.).

3.2. Volatility measures

Variance. The variance and the closely-related standard deviation are measures of how spread out a distribution is. The variance is computed as the average squared deviation of each number from its mean:

$$s_2 = \sum_{i=1}^{m} p_1(R_i - R)^2, \qquad (3.2.1)$$

where:
R – expected return,
R_i – actual return,
p_i – probability of an i-element rate of return.

Standard deviation. The standard deviation (the square root of the variance) is the most commonly used and the most important measure of variability. Standard deviation uses the mean of the distribution as a reference point and measures variability by considering the distance between each score and the mean.

In simple terms, the standard deviation provides a measure of the standard, or average, distance from the mean, and describes whether the scores are clustered closely around the mean or are widely scattered. The standard deviation, is a measure of how far the actual return is likely to deviate from the expected return (see Gravetter, Wallnau, 2010, p. 91, and Brigham, Houston 2011, p. 264).

3.3. Downside risk measures

Semivariance and standard semideviation. Markowitz proposed semivariance as an alternative measure of risk. Semivariance is the same as variance, except that the riskiness (as measured by a typical deviation from the average return) is calculated using only the points below the mean (por. Adams et al 2003, s. 145).

$$sv = \sum_{i=1}^{m} p_1 d_i^2, \qquad (3.3.1)$$

$$d_i = \begin{cases} R_i - R, & \text{when } R_i - R < 0 \\ 0, & \text{when } R_i - R \geq 0. \end{cases}$$

Standard semideviation is the square root of the semivariance which equals to the doubled area of the downside dispersion space.

Average negative deviations from the objective value (generalized negative semivariance). Standard semideviation and the semivariance do not measure deviations from a particular objective value, but from the mean value. As a rule, these two values are not identical. This point of criticism may be omitted in the case of level target, since a negative or alternatively positive generalized semivariance can be used instead of the semivariance In the generalized semivariance formula the expected value is replaced with a given value of objective:

$$\sigma\_ (R) = - \sum_{i=1}^{m} d_i^- \cdot p_i, \qquad (3.3.2)$$

where:

$$d_i^- = \begin{cases} R_i - R_0, & \text{when } R_i - R_0 < 0 \\ 0, & \text{when } R_i - R_0 \geq 0. \end{cases}$$

and:

R_0 – objective's value,
R_i – actual return,
p_i – probability of an i-element rate of return.

3.4. Risk coefficient

T. Zaleśkiewicz claims that 'according to the rules of decisive analysis, when assessing the risk volume, four parameters should be taken into consideration: (1) loss volume, (2) loss probability, (3) profit amount and (4) profit probability' (Zaleśkiewicz 2004, p. 90). The comprehension of positive deviations is also of great significance from the point of view of legal assessment of risk. The legally acceptable risk should be distinguished from unlawful danger. Therefore, the relations between risk, costs and benefits need to be analyzed very carefully (Zellmer 1990, p. 16).

This is the reason why the risk coefficient is considered to be the most effective risk measure in the case of level targets. The risk coefficient is a quotient of generalized negative semivariance (average negative deviations from the objective value) and generalized positive semivariance (average positive deviations from the objective value). (Báskai et al 1979, p. 69, Zellmer 1990, p. 52). The lower the coefficient, the lower the risk. (Zellmer, Wasilewski 2010, p. 401). The risk coefficient r_c is given by the formula:

$$r_c(R) = \frac{\sigma_-(R)}{\sigma^+(R)}, \qquad (3.4.1)$$

where:

$$\sigma_+(R) = \sum_{i=1}^{m} d_i^+ \cdot p_i,$$

and:

$$d_i^+ = \begin{cases} 0, & \text{when } R_i - R_0 \leq 0 \\ R_i - R_0. & \text{when } R_i - R_0 > 0. \end{cases}$$

and:
R_0 – objective's value,
R_i – actual return,
p_i – probability of an i-element rate of return.

4. Case study

4.1. Basic assumptions

Production program of a company should be extended for another product. Manager can choose from four products (V1-V4). Evaluation of variants of the investments is made by the profit criterion (criterion of maximizing the rate of return). The profit depends on market conditions, which are not foreseeable. Therefore, when making the decision, six possible market situations will be taken into consideration:
S1 – very bad market situations,
S2 – bad market situations,

S3 – difficult market situations,
S4 – average market situations,
S5 – good market situations,
S6 – very good market situations.
For all market situations the experts have estimated probabilities p_j of these types of economic trends. A decision matrix is as follows:

Table 4.1.1. Decision matrix

	S1	S2	S3	S4	S5	S6
V1	40	25	30	35	45	40
V2	20	40	35	35	40	50
V3	40	45	55	35	35	40
V4	30	35	35	45	50	35
p_j	0,1	0,2	0,25	0,2	0,15	0,1

Source: own study

Which option should be selected (V1-V4) when we know that the company's objective is the rate of return in minimum size of 40?

4.2. Determination of volatility measures

Estimating of of risk measures occurs in computer program R-project for statistical computing. The algorithm of data entry is as follows:
We enter data as vectors

```
a=c(40,25,30,35,45,40)
b=c(20,40,35,35,40,50)
c=c(40,45,55,35,35,40)
d=c(30,35,35,45,50,35)
x=c(0.1,0.2,0.25,0.2,0.15,0.1)
```

in order to make further calculations it is necessary to set the **expected value** for each variant:

```
ea=sum(a*x)
> ea
[1] 34.25
```

```
> ec=sum(c*x)
> ec
[1] 43
```

```
> eb=sum(b*x)
> eb
[1] 36.75
```

```
> ed=sum(d*x)
> ed
[1] 38.75
```

In order to determine the variance and standard deviation of the rate of return we use the formula [3.2.1.]

```
skw_a=sum(x*(a-ea)^2)
> skw_a
[1] 45.6875
> skw_b=sum(x*(a-eb)^2)
> skw_b
[1] 51.9375
> skw_c=sum(x*(a-ec)^2)
> skw_c
[1] 122.25
> skw_d=sum(x*(a-ed)^2)
> skw_d
[1] 65.9375
```

```
sa=sqrt(skw_a)
> sa
[1] 6.759253
> sb=sqrt(skw_b)
> sb
[1] 7.206768
> sc=sqrt(skw_c)
> sc
[1] 11.05667
> sd=sqrt(skw_d)
> sd
[1] 8.120191
```

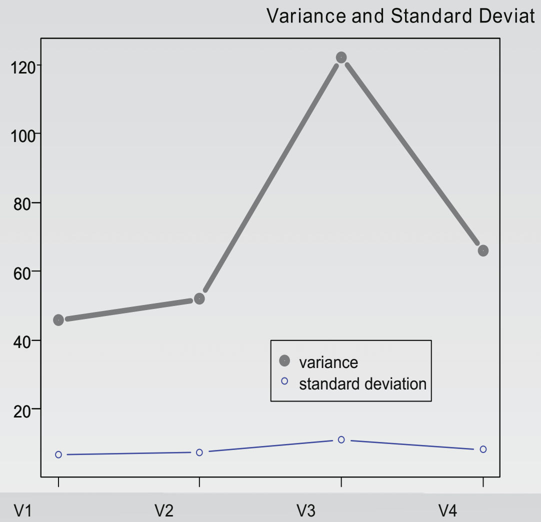
Table 4.2.1. Values of variance and standard deviation for the four variants

	s^2	s
V1	45,69	6,76
V2	51,94	7,2
V3	122,25	11,06
V4	65,94	8,12

Source: own study

According to the criterion of variance (deviation) manager should choose the variant V1 since it is the smallest of its value. Deviation of the actual value from the expected rate of return are the smallest.

Figure. 4.2.1. Illustration of the variance and standard deviation



Source: own study

4.3. Determination of downside risk measures

In order to determine the **semivariance and standard semideviation** of the rate of return we use the formula [3.3.1.]

```
> sva=sum(x*d_a^2)
> sva
[1] 21.62813
> svb=sum(x*d_b^2)
> svb
[1] 29.43438
> svc=sum(x*d_c^2)
> svc
[1] 24.2
```

```
> svd=sum(x*d_d^2)
> svd
[1] 15.39062

> ssa=sqrt(sva)
> ssa
[1] 4,65
> ssb=sqrt(svb)
> ssb
```

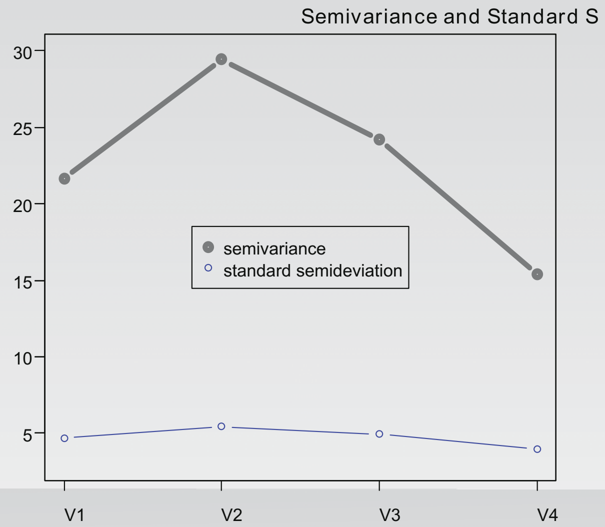
```
[1] 5,42
> ssc=sqrt(svc)
> ssc
[1] 4,92
```

Table 4.3.1. Values of semivariance and standard semideviation

	sv	ss
V1	21.63	4,65
V2	29.43	5,42
V3	24.2	4,92
V4	15.39	3,92

Source: own study

Figure. 4.3.1. Illustration of semivariance and standard semideviation



Source: own study

As for the interpretation of the variance and standard deviation, the best option is the one with the lowest values of these parameters. Manager should also be aware that these measures are based on the meaning of risk as a negative phenomenon. To determine the risk is here exclusively negative deviations from the expected rates of return. By this measure V4 wins.

In order to determine the **average negative deviations from the objective value** we use the formula [3.3.2.]

```
s_minus_a
[1] 6.5
> s_minus_b
[1] 4.25
> s_minus_c
[1] 1.75
> s_minus_d
[1] 3.75
```

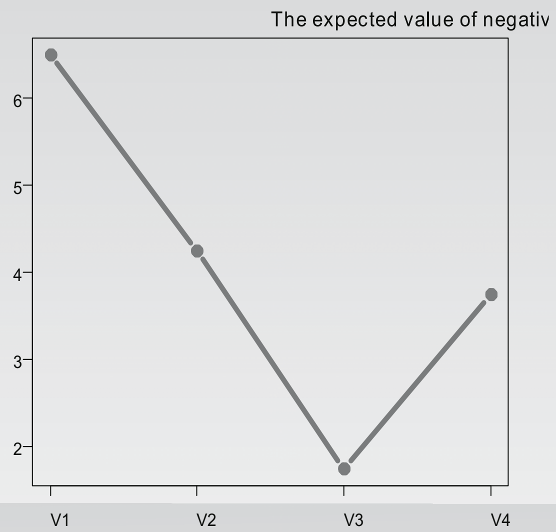
Table 4.3.2. Average negative deviations from the objective value

	σ_-
V1	6.5
V2	4.25
V3	1.75
V4	3.75

Source: own study

The third variant has the smallest average deviation from the objective, therefore, according to this criterion it should be preferred by decision-makers. This means that this product guarantees the rate of return which is the nearest of objective and also guarantees the least possible risk of loss. It should be noted, that this coefficient illustrates only negative deviations from the objective, without the opportunity to overrun. Positive deviation and associated benefits also play a role in the process of investing. The measure, which takes into account both situations is called risk coefficient (calculated below)

Figure. 4.3.2. Illustration of an average negative deviations from the objective value



Source: own study

In order to determine the **risk coefficient** we use the formula [3.4.1.]

```
rk_a=(s_minus_a)/(s_plus_a)
> rk_a
[1] 8.666667
> rk_b=(s_minus_b)/(s_plus_b)
> rk_b
[1] 4.25
> rk_c=(s_minus_c)/(s_plus_c)
> rk_c
[1] 0.3684211
> rk_d=(s_minus_d)/(s_plus_d)
> rk_d
[1] 1.5
```

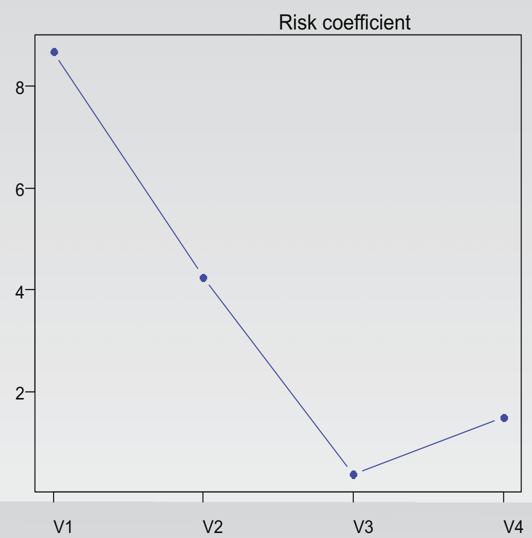
The option, with a minimum value of this parameter will be preferred. The third variant has the smallest risk coefficient. Negative deviations from the objective are relatively much smaller than the positive deviations. This can be seen in the decision matrix. In situation S3 the rate of return is up to 55 and therefore about 15 more than the minimum.

Table 4.3.3. Values of risk coefficient

	r_c
V1	8.666667
V2	4.25
V3	0.3684211
V4	1.5

Source: own study

Figure. 4.3.3. Illustration of risk coefficient values



Source: own study

5. Conclusion

The only measure which relates to the established objective (a concrete rate of return) and not just the expected value, and includes both positive and negative deviations from that objective, is the proposed risk coefficient. As we can see in the decision matrix, none of variants guarantee objective's achievement in all situations. Therefore, the choice of each of them involves the risk of non-achievement of an objective. In this example, the third variant has the smallest investment risk. He obtained the best result (given the least risk) in the two most important parameters (risk coefficient and The expected negative deviations from the objective). Variant V1 was the best under the criterion of variance and standard deviation. However, under the criterion of semivariance and standard semideviation as the best proved to be variant V4. On the basis of the results of risk assessments, the manager may decide about the type of investment.

Performing calculations using the provided formulas can be difficult and can cause technical difficulties for the management and low-skilled workers. These skills can be acquired in the process of training and retraining of workers.

More important problem seems to be the availability of data needed to computing. In many cases, there are serious difficulties related to gathering historical data, to assist in the extrapolation of future values. In most cases, the Executive team is faced with the problem of correct estimation of the rates of return on investments and to determine the value of the probability. If this issue will not be resolved successfully, you will not be able to effectively apply these measures of risk. Although the risk characterization, and lack of data poses many difficulties, the benefits of these methods outweigh the inputs.

Summary

Application of quantitative metrics for assessing the investment risk

Identification and assessment of risk-especially in small and medium-sized enterprises – from the beginning were the part of the topics of research scientists. Significant investments are also related to the implementation and correct application of quantitative measures of risk assessment. The article compares the important quantitative measures of risk and presents their practical calculation. The paper shows the correct interpretation of the results of calculations, that should be helpful in decision-making by managers.

Streszczenie

Zastosowanie miar ilościowych przy ocenie ryzyka inwestycyjnego

Identyfikacja i ocena ryzyka – zwłaszcza w małych i średnich przedsiębiorstwach – należały od początku do tematów badawczych naukowców. Znaczące są również nakłady związane z wdrożeniem i poprawnym zastosowaniem ilościowych miar oceny ryzyka.

Artykuł porównuje najważniejsze ilościowe miary ryzyka oraz prezentuje ich praktyczne obliczenie. W pracy przedstawiono poprawną interpretację wyników obliczeń, która winna być pomocna przy podejmowaniu decyzji przez managerów.

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